

A Note on the Misuse of the Variance Test in Meteorological Studies

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Abstract

The erroneous assumption “for all distributions for which the theoretical variance can be computed independently from parameters estimated by any method different from the method of moments” has been used in the case of fitting the gamma distribution to a rainfall data by Mooley (1973) which was followed by several researchers. We show that the asymptotic distribution of the test statistic is generally not even comparable to any central chi-square distribution. We also describe a method for checking the validity of the asymptotic distribution for a class of distributions.

Keywords: *Asymptotic theory; Chi-square test; P-value; Null distribution; Rainfall data; Variance ratio test.*

1. INTRODUCTION

The variance ratio test statistic provides a measure of goodness-of-fit. In the spirit of the pioneering idea of Fisher (1925) as an index of dispersion, Cochran (1954) efficiently used and popularized this test, illustrating with examples in the case of small samples from the Poisson and the Binomial series. The test statistic was referred to as the central chi-square distribution with degrees of freedom one less than the sample size in both the cases, whether the parameters are specified or not, and a proof of this fact was given by Rao and Chakravarti (1956) for the Poisson series. For large sample size, a modified form of the test statistic was proposed by Fisher and Yates (1957) so that its asymptotic distribution corresponds to the standard normal density.

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Perhaps due to ignorance related to the asymptotic theory of the variance ratio test statistic, Mooley (1973) committed a misuse of this goodness-of-fit test in the case of fitting a gamma distribution to Asian summer monsoon rainfall data, assuming that the test can be used for all distributions for which the theoretical variance can be computed independently of parameters estimated by a method other than the method of moments. He also assumes that the asymptotic distribution of the test statistic in such a situation would be a chi-square distribution with degrees of freedom one less than the sample size. A significant number of other authors (Hargreaves (1975), Sarker et al. (1982), Biswas et al. (1989), Goel and Singh (1999)) followed the similar misuse synergistic with the work of Mooley (1973).

As far our knowledge is concerned, no potential work has yet explored the fact that the implementation of the variance ratio test by Mooley (1973) was incorrect. Several studies have been conducted on rainfall analysis and the best fit probability distribution function such as the gamma distribution function (Barger and Thom (1949), Mooley and Crutcher (1968), Sen and Eljadid (1999)), log-normal (Sharma and Singh (2010), Kwaku et al. (2007)), exponential (Duan et al. (1995), Burgueno et al. (2005), Todorovic and Woolhiser (1975)), Weibull (Duan et al. (1995), Burgueno et al. (2005)) distributions were identified under different situations. Our simulation study suggests that when the data sets have the proximity to any one of exponential, gamma, Weibull, log-normal, the usual asymptotic distribution of the test statistic is no longer even central chi-square and thus the variance ratio test can not be used under any of the circumstances. Below we provide a brief overview of the issues involved.

1.1 Variance ratio test and its misuse

Suppose we want to fit the random sample X_1, \dots, X_n to a distribution whose cumulative distribution function is given by F . We consider the following hypothesis testing problem – H_0 : the sample comes from the distribution F , versus H_1 : the sample does not come from the distribution F . The test statistic proposed by Fisher (1925) and illustrated by Cochran (1954) is

$$\chi_\nu^2 = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\widehat{\sigma_F^2}},$$

where $\widehat{\sigma_F^2}$ is the estimate of the population variance computed independently of parameters estimated by a method other than the method of moments. So, this method is not applicable to fitting distributions like normal. The test statistic was used in the case of Poisson and Binomial distributions by Fisher (1925) but no proper mathematical justification was provided. Cochran (1936) and Rao and Chakravarty (1956) calculates a form of the approximate expression of the mean and variance of the distribution of the test statistic and also provide justification of implementing this test for Poisson and Binomial distributions under the null hypothesis.

Mooley (1973) uses this test for fitting gamma distribution to the monsoon rainfall data and estimates the unknown parameters using maximum likelihood estimation. Thus, the estimate of the population variance and the sample variance do not coincide. Mooley (1973) states that the method works well for any probability model satisfying such criterion. In Section 2 we provide theoretical justification why the statement is incorrect, demonstrating the issue with the exponential distribution; we also conduct a simulation study, justifying the same with several other distributions – gamma, log-normal and Weibull. In Section 3 we identify a large class of distributions where we can simply check whether or not the usual asymptotics as in the cases of Poisson and Binomial are valid. In particular, we discuss and derive the asymptotics of the variance ratio test for a large class of distributions with finite fourth moment when the population variance can be written as a differentiable function of the population mean.

2. ANALYTICAL AND EMPIRICAL EXPOSITION OF THE MISUSE OF THE VARIANCE RATIO TEST

Suppose that X_1, X_2, \dots, X_n is a random sample of size n from the exponential distribution with mean λ , where the density is given by

$$f(x) = \frac{1}{\lambda} e^{-\frac{x}{\lambda}}.$$

In the above, the mean λ is unknown, which we assume to be estimated from the sample using the maximum likelihood estimation (MLE) procedure.

With the above set-up, Theorem 1 shows that the asymptotic distribution of the variance ratio

test statistic is not χ_{n-1}^2 .

Theorem 1 *The asymptotic distribution of the variance ratio test statistic, under the null hypothesis that a random sample of size n comes from a one-parameter exponential distribution, is not comparable with central χ^2 distribution with $n - 1$ degrees of freedom.*

Proof. The estimate of the unknown parameter λ is $\lambda_{MLE} = \frac{X_1 + X_2 + \dots + X_n}{n} = \bar{X}$, that is, the sample mean. The population variance is λ^2 and thus, the MLE of the population variance is $\hat{\lambda}_{MLE}^2 = (\hat{\lambda}_{MLE})^2 = \bar{X}^2$. Hence, the test statistic in our case is given by

$$D = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\bar{X}^2}.$$

For the remaining part of the proof we follow Rao and Chakravarti (1956) who provide justification of the asymptotic properties of the variance ratio test in the case of the Poisson distribution, but make necessary modifications to accommodate our case of continuous distribution.

We note that the sample total given by $T = X_1 + X_2 + \dots + X_n$ is sufficient for λ . Here T follows the gamma distribution with shape parameter n and scale parameter λ ; the density is given by

$$f_T(t) = \frac{1}{\lambda^n \Gamma(n)} e^{-\frac{t}{\lambda}} t^{n-1}.$$

The conditional density of X_1, X_2, \dots, X_n given $T = t$ is given by

$$f_{X_1, X_2, \dots, X_n | T=t}(x_1, x_2, \dots, x_n) = \frac{\Gamma(n)}{t^{n-1}}.$$

Now, $E(X_i | T) = \bar{X} = T/n$ and thus we can express the variance ratio test statistic in the form

$$D = \sum_{i=1}^n \frac{(X_i - E(X_i | T))^2}{E(X_i | T)^2}.$$

Now, by the definition of conditional expectation, we have, for any measurable function $\phi(x_1, x_2, \dots, x_n)$:

$$\int_0^\infty E(\phi | T = t) f_T(t) dt = E(\phi),$$

which, in our case, translates into

$$\int_0^\infty E(\phi|T=t)e^{-\frac{t}{\lambda}}t^{n-1}dt = E(\phi)\lambda^n\Gamma(n).$$

Therefore, knowing the total expectation $E(\phi)$, the conditional expectation $E(\phi|T=t)$ can be easily obtained. Let us consider the statistic $S^2 = \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - n\bar{X}^2$ whose moments are known functions of λ . Using the above definition of conditional expectation we derive the conditional moments of $\phi(x_1, x_2, \dots, x_n) = S^2$ as follows.

Since $E(S^2) = (n-1)\lambda^2$, we have

$$\int_0^\infty E(S^2|T=t)e^{-\frac{t}{\lambda}}t^{n-1}dt = (n-1)\lambda^{n+2}\Gamma(n).$$

Now, we can write $\lambda^{n+2} = \int_0^\infty \frac{1}{\Gamma(n)}e^{-\frac{t}{\lambda}}t^{n+1}dt$, and thus it follows that

$$\int_0^\infty E(S^2|T=t)e^{-\frac{t}{\lambda}}t^{n-1}dt = \int_0^\infty \frac{(n-1)\Gamma(n)}{\Gamma(n+2)}e^{-\frac{t}{\lambda}}t^{n+1}dt.$$

We know that if $\int_0^\infty f_1(x)e^{-ax}dx = \int_0^\infty f_2(x)e^{-ax}dx$ where $f_1(x), f_2(x)$ and both are continuous, a is some positive constant, then, $f_1 = f_2$ by the uniqueness of the Laplace transform. As a consequence,

$$E(S^2|T=t) = \frac{(n-1)}{n(n+1)}t^2,$$

and hence

$$\begin{aligned} E(D|T=t) &= E(S^2 \frac{n^2}{T^2} | T=t) \\ &= \frac{n^2}{t^2} \frac{(n-1)}{n(n+1)} t^2 \\ &= \frac{(n-1)n}{n+1} \\ &\approx n-1. \end{aligned}$$

Similarly, we obtain

$$E(S^4) = \frac{(n-1)(n^2+7n-6)}{n}\lambda^4$$

and

$$E(S^4|T = t) = \frac{(n-1)(n^2+7n-6)\Gamma(n)}{n\Gamma(n+4)}t^4.$$

Thus,

$$\begin{aligned} E(D^2|T = t) &= E(S^4 \frac{n^4}{T^4}|T = t) \\ &= \frac{n^4}{t^4} \frac{(n-1)(n^2+7n-6)\Gamma(n)}{n\Gamma(n+4)}t^4 \\ &= \frac{n^2(n-1)(n^2+7n-6)}{(n+3)(n+2)(n+1)}, \end{aligned}$$

and so,

$$\begin{aligned} Var(D|T = t) &= E(D^2|T = t) - (E(D|T = t))^2 \\ &= 4(n-1) \frac{1}{(1 + \frac{1}{n})^2(1 + \frac{2}{n})(1 + \frac{3}{n})} \\ &\approx 4(n-1). \end{aligned}$$

Since $Var(D|T = t)$ is independent of t , it follows that $Var(D) = Var(D|T = t)$, which does not conform with the variance of the central chi-square distribution with $(n-1)$ degrees of freedom which is $2(n-1)$. This proves the theorem.

■

2.1 Simulation study to demonstrate the effect of the erroneous assumption

To demonstrate the effect of the erroneous assumption of χ_{n-1}^2 as the asymptotic distribution of D , we consider a simulation study pertaining to the cases of exponential, gamma, log-normal and Weibull. We calculate the values of the empirical mean and empirical variance for different values of the parameters for different null distributions, based on 10,000 simulated samples in each case.

The results are presented in Table 2.1. Correct usage of Cochran's variance ratio test should yield the mean and the variance close to 100 and 200 respectively in case (a) and 200 and 400

respectively in case (b). However, the results in Table 2.1 are far from the aforementioned values, clearly pointing towards incorrect implementation of the test.

3. CHECKING THE VALIDITY OF THE χ^2 ASSUMPTION FOR THE ASYMPTOTIC DISTRIBUTION OF THE VARIANCE RATIO TEST STATISTIC

Theorem 2 below provides a way to check the validity of the χ^2_{n-1} assumption for the asymptotic distribution of D .

Theorem 2 *If a random sample of size n comes from a population with finite fourth moment where the population variance is a differentiable function f of the population mean under the null hypothesis, then under the condition (which we refer to as the “condition of approximate equality”)*

$$\frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2\mu_3 f'(\mu) + f(\mu)^2(\mu_4 - \sigma^4)) \approx 2,$$

where $\mu, \sigma^2, \mu_3, \mu_4$ are the mean, 2nd, 3rd and 4th central moments of the population respectively, the variance ratio test statistic is asymptotically central χ^2 with $n - 1$ degrees of freedom. If a function like f exists and the “condition of approximate equality” fails, then the variance ratio test statistic is not asymptotically χ^2_{n-1} .

Proof. Suppose that X_1, X_2, \dots, X_n is a random sample from a population where the sufficient condition on moment existence and the existence of a differentiable function f are satisfied under the null hypothesis.

Applying the bivariate central limit theorem (CLT) in the context of sample moments, we obtain the joint asymptotic distribution of sample mean \bar{X}_n and sample variance S_n^2 as

$$\sqrt{n} \left[\begin{pmatrix} X_n \\ S_n^2 \end{pmatrix} - \begin{pmatrix} \mu \\ \sigma^2 \end{pmatrix} \right] \rightarrow N_2 \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \mu_3 \\ \mu_3 & \mu_4 - \sigma^4 \end{pmatrix} \right) \text{ in distribution,} \quad (3.1)$$

where μ is the population mean, σ^2 is the population variance, μ_3 and μ_4 are the third and the fourth central moments of the population, respectively.

In the case of asymptotic normality of smooth functions of sample moments, it was shown by Cramer (1946) that for a mapping $\mathbf{g} : \mathbb{R}^d \rightarrow \mathbb{R}^k$ such that $\mathbf{g}'(\mathbf{x})$, the derivative of $\mathbf{g}(\mathbf{x})$ at the point \mathbf{x} , is continuous in a neighborhood of $\boldsymbol{\theta} \in \mathbb{R}^d$, if \mathbf{T}_n is a sequence of d -dimensional random vectors such that $\sqrt{n}(\mathbf{T}_n - \boldsymbol{\theta}) \rightarrow \mathbf{N}_d(\mathbf{0}, \boldsymbol{\Sigma})$ where $\boldsymbol{\Sigma}$ is a $d \times d$ covariance matrix, then

$$\sqrt{n}(\mathbf{g}(\mathbf{T}_n) - \mathbf{g}(\boldsymbol{\theta})) \rightarrow \mathbf{N}_k(\mathbf{0}, \mathbf{g}'(\boldsymbol{\theta})\boldsymbol{\Sigma}\mathbf{g}'(\boldsymbol{\theta})^T) \quad \text{in distribution.} \quad (3.2)$$

In our case, the population mean is estimated by the sample mean, and since $\sigma^2 = f(\mu)$, the population variance is estimated by $f(\bar{X}_n)$ which is neither equal nor proportional to S_n^2 (otherwise σ^2 can not be written as a function of μ only). Hence $D = \sum_{i=1}^n \frac{(X_i - \bar{X}_n)^2}{f(\bar{X}_n)} = n \frac{S_n^2}{f(\bar{X}_n)}$ can be used as a test statistic. Then using the delta method (3.2) associated with (3.1), we have

$$\sqrt{n} \left(\frac{S_n^2}{f(\bar{X}_n)} - \frac{\sigma^2}{f(\mu)} \right) \rightarrow N \left(0, \frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2\mu_3 f'(\mu) + f(\mu)^2(\mu_4 - \sigma^4)) \right) \quad (3.3)$$

in distribution.

Now, if

$$\frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2\mu_3 f'(\mu) + f(\mu)^2(\mu_4 - \sigma^4)) \approx \alpha,$$

where $\alpha = 2$, then the asymptotic distribution (3.3) is $N(n, 2n)$. Since $N(n, 2n) \approx \chi_{n-1}^2$, in this case the variance test statistic D is asymptotically distributed as χ_{n-1}^2 .

On the other hand, if α is significantly different from 2, then $E(D) = n$ but $Var(D) \neq 2(n-1)$, even asymptotically. Hence, the asymptotic distribution of D can not be central χ_{n-1}^2 in this case.

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The following examples can be viewed as corollaries to Theorem 2.

3.1 Illustrative examples

3.1.1. Poisson case In the case of Poisson distribution with parameter λ , the delta method with $g(x, y) = \frac{y}{f(x)}$ and $f(x) = x$ yields

$$\frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2\mu_3 f'(\mu) + f(\mu)^2(\mu_4 - \sigma^4)) = 2.$$

Hence, $D = \sum_{i=1}^n \frac{(X_i - \bar{X}_n)^2}{\bar{X}_n} = n \frac{S_n^2}{\bar{X}_n}$ has asymptotic distribution $N(n, 2n) \approx \chi_{n-1}^2$.

3.1.2. Binomial case In the case of Binomial distribution with size M and probability p , applying the delta method with $g(x, y) = \frac{y}{f(x)}$ and $f(x) = \frac{x(M-x)}{M}$, we obtain

$$\frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2 \mu_3 f'(\mu) + f(\mu)^2 (\mu_4 - \sigma^4)) = 2 + \frac{1}{M}.$$

Since, for large enough M , $\frac{1}{M} \approx 0$, $D = \sum_{i=1}^n \frac{(X_i - \bar{X}_n)^2}{\bar{X}_n(M - \bar{X}_n)} = n \frac{S_n^2}{\bar{X}_n(M - \bar{X}_n)}$ has $N(n, 2n) \approx \chi_{n-1}^2$ as the asymptotic distribution.

3.1.3. Exponential case In the case of exponential distribution with mean λ , let $g(x, y) = \frac{y}{f(x)}$ and $f(x) = x^2$. The delta method then yields

$$\frac{1}{f(\mu)^4} (\sigma^6 (f'(\mu))^2 - 2\mu\sigma^2 \mu_3 f'(\mu) + f(\mu)^2 (\mu_4 - \sigma^4)) = 4.$$

Hence, $D = \sum_{i=1}^n \frac{(X_i - \bar{X}_n)^2}{\bar{X}_n^2} = n \frac{S_n^2}{\bar{X}_n^2}$ has the asymptotic distribution $N(n, 4n)$, which can not be approximated by χ_{n-1}^2 . So, the test can not be used for exponential distributions. Note that this method of validation provides a straightforward way of proving Theorem 1.

4. RELEVANCE OF THE STUDY IN RAINFALL DATA

Using data from 39 well-distributed and long-record stations over a relevant study region, and implementing the χ^2 goodness-of-fit test, the Kolmogorov-Smirnov test and the variance ratio test, Mooley (1973) found the two-parameter gamma distribution to be the most suitable probability model among the Pearsonian models that show good fit to monthly rainfall in the Asian summer monsoon. After implementing the variance ratio test in the context of weekly rainfall total, Hargreaves (1975) obtained the two-parameter incomplete gamma distribution suitable for the modeling purpose. Sarker et al. (1982) computed the lowest amount of rainfall in the dry farming tract of north-west and south-west India at different probability levels by fitting the same probability model, which was obtained by implementing the same variance ratio test. On the basis of the same model they also considered 50% probabilistic rainfall as dependable precipitation on a weekly basis. Biswas and Khambete (1989) computed the lowest amount of rainfall at different probability

levels by fitting the same model, which was obtained by implementing the same variance ratio test on a data regarding week by week total rainfall of 82 stations in dry farming tract of Tamilnadu state of south-east India. Goel and Singh (1999) fitted the weekly rainfall data of Soan catchment in sub-humid area of Shivalik region of northern India to the same model, which they obtained by implementing the same test.

5. EFFECTS OF THE ERRONEOUS ASSUMPTION BY MOOLEY (1973) ON INFERENCE: ILLUSTRATIONS WITH SIMULATED AND REAL DATA

Among the 39 Rain gage stations considered in Mooley (1973), the null hypothesis that the monthly rainfall series follow gamma distribution, was rejected for three cases. In particular, the null hypothesis associated with the September rainfall of Allahabad, India, and July rainfall of Zi-Ka-Wei, China, were both rejected at level 0.05, using the variance ratio test. The test statistic in the case of June rainfall of Nagpur, India, was found to be significant at level 0.01. In case of the χ^2 test, the null hypothesis was accepted at level 0.05 for all the considered cases.

For adequate investigation of the above results obtained by Mooley, the actual data set used in Mooley (1973) is necessary. But unfortunately the data set is unavailable. As a result, we are compelled to conduct further simulation studies to demonstrate that Mooley's implementation can lead to rejection of the correct null hypothesis and acceptance of the false null hypothesis with high probability. However, in Section 5.2 we also investigate the effects of Mooley's erroneous assumption using a real data set obtained from an independent source.

5.1 Simulation based illustration of false rejection and false acceptance of the null hypothesis using Mooley's implementation

5.1.1. First simulation study: false rejection of H_0 First we draw 100,000 samples of size 100 from the gamma distribution with scale parameter $\lambda = 2$ and shape parameter $\alpha = 0.5$; the histogram of the observed test statistic is presented in Figure 5.1. Now, according to the claim of Mooley (1973), the test statistic should be distributed as χ^2 with degrees of freedom $100 - 1 = 99$. We draw the cut-offs as the vertical lines for a goodness of fit test of level 0.05. As we draw

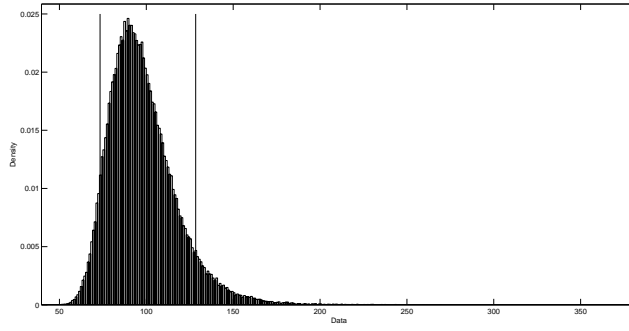


Figure 5.1: Histogram of the null distribution of the test statistic; the vertical lines indicate the cut-off levels according to Mooley (1973).

samples from the null hypothesis, the expected number of rejections should be 5000. But here we see that the number of rejections is 13,214, which is far above than the expected number of rejections. Thus, this experiment demonstrates that there is a high chance of rejection of the null hypothesis even if the sample actually arises from the distribution under H_0 .

5.1.2. Second simulation study: false acceptance of H_0 We conduct another simulation study where we simulate 100,000 samples of size 30 from a mixture of three gamma distributions with equal weight (that is, each mixture component has mixing probability $\frac{1}{3}$). For the three gamma components, the parameters were chosen in such a way that the modes of the components are 1, 5 and 9 respectively, while the variance under each component is specified to be 1. The true mixture density, depicted in Figure 5.2, is clearly significantly different from any single gamma distribution.

According to the claim of Mooley (1973), the number of cases of the rejection of the null hypothesis should be large enough, much more than 5%, that is, 5000 cases. But in our simulated example only 3470 cases were rejected, even much less than the expected number of rejections under the null (the cases lying outside the cut-offs are shown in Figure 5.3). This experiment thus demonstrates that this test may often lead to false acceptance of the null hypothesis that the data is distributed as gamma while in reality the actual distribution is very far from gamma.

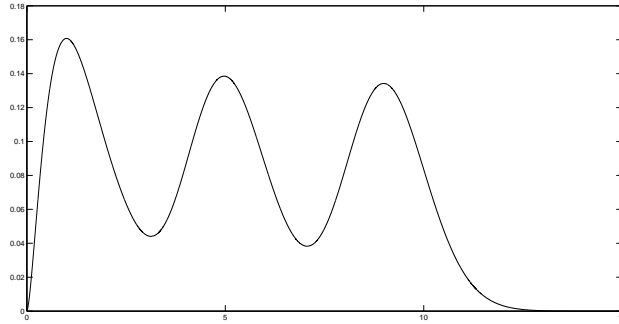


Figure 5.2: Density of the mixture of three gamma distributions: significantly different from gamma.

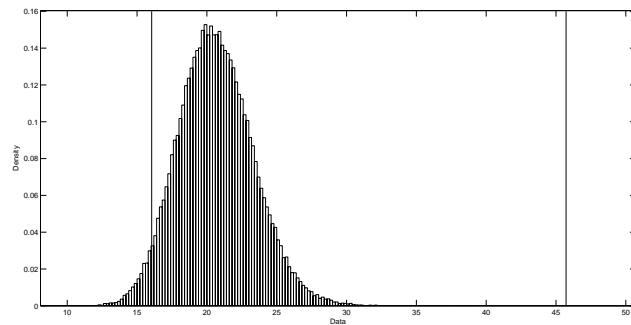


Figure 5.3: Histogram of the null distribution of the test statistic; the vertical lines indicate the cut-off levels according to Mooley (1973).

Next, we illustrate the issue of false rejection and false acceptance of the null hypothesis under Mooley's implementation with a real data set.

5.2 Illustration with June-September rainfall of India

We obtain the dataset of All India Seasonal Rainfall Series (1901-2009) from the website of India Meteorological Department (<http://www.imd.gov.in/section/nhac/dynamic/data.htm>). In Figure 5.4, we present the histogram of the observed dataset to which we fit a gamma distribution. For the χ^2 goodness-of-fit test the P -value turns out to be 0.0167, that is, for a test of level 0.05, we reject the null hypothesis that the data is distributed as gamma. From Figure 5.4, it is also evident that the fit is not “good”. Now, using the variance test in this set-up, the MLEs of the shape and scale parameters are 9.8663 and 91.0873 respectively, and the observed variance test statistic is 107.2916. Assuming that the claim of Mooley (1973) about the asymptotic null distribution of the variance test statistic is true, the P -value turned out to be

$$P(|Z| > |\sqrt{2 \cdot 107.2916} - \sqrt{217}|) = 0.9344,$$

where $Z \sim N(0, 1)$, leading to acceptance of the gamma distribution. However, poor fit exhibited by Figure 5.4, rejection of the gamma distribution by the formal χ^2 test, and wisdom gained from our analytical and simulation based investigations regarding Mooley's implementation strongly suggests that this variance test wrongly accepts the false null hypothesis.

6. DISCUSSION AND CONCLUSION

The increased power of the variance test over Pearson's chi-square goodness-of-fit test was strikingly shown in some sampling experiments conducted by Berkson (1940), in a situation where the data followed a Binomial distribution. Berkson (1938) presented some data to illustrate the cases where the variance ratio test was significant but Pearson's chi-square goodness-of-fit test was not, in the contexts where the data followed a Poisson or a Binomial distribution.

However, when the underlying data follow a two-parameter gamma distribution, the asymptotic distribution of the variance ratio test statistic is largely dependent on the shape parameter and, as

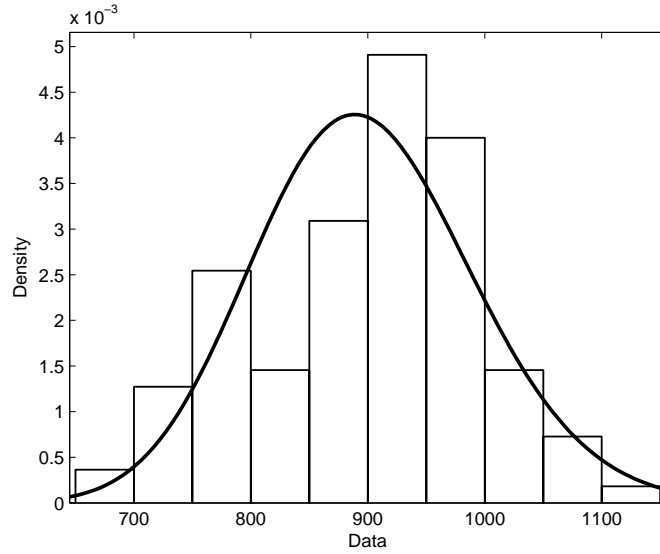


Figure 5.4: Histogram of the observed dataset and the fitted gamma density.

a consequence, the assumption that the test statistic asymptotically follows a central chi-square distribution, is erroneous and leads to misuse of the variance ratio test. Mooley (1973) seems to be the first to commit this misuse and a significant number of other authors followed the same path leading to misuse. Indeed, as we have shown in this article, for probability distributions like exponential, log-normal, Weibull, etc., which are frequently used in modelling rainfall data, the asymptotic distribution of the variance ratio test statistic is not commensurate with the chi-square distribution with degrees of freedom $n - 1$. Hence, the test should be used very cautiously, particularly by meteorologists and other scientists.

To aid the meteorologists and the other practising scientists, in this article we have provided simple ways to check the validity of the variance ratio test for a large class of distributions satisfying a few mild conditions. In fact, as a necessary condition for applicability of the test, first it should be checked whether the limiting mean and variance are comparable with $n - 1$ and $2(n - 1)$ respectively.

If the variance ratio test is not applicable, it is better to use chi-square goodness-of-fit test in spite of having less power and loss of information by clubbing the data into different classes. At least it is theoretically correct and can be used in the case of fitting a mixture of zero rainfall and

non-zero rainfall data. In the case of fitting non-zero rainfall data, it is more appropriate to use the Kolmogorov-Smirnov test than chi-square goodness-of-fit test in cases where the parameters under the null hypothesis are fully specified.

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Table 2.1: Table of means and variances of the variance ratio test statistic: (a) sample size 100, (b) sample size 200 for different distributions

			Parameter values				
			1	5	10	15	20
Exponential (mean=parameter)	(a)	mean	97.68	97.79	98.15	97.92	97.92
		variance	364.64	360.17	373.09	370.65	362.63
	(b)	mean	197.37	197.96	198.07	197.99	198.48
		variance	756.86	754.43	765.76	765.13	786.41
Gamma (scale=2, shape=parameter)	(a)	mean	98.95	99.78	99.97	99.96	99.93
		variance	220.38	48.69	24.45	16.73	12.22
	(b)	mean	199.04	199.95	199.94	199.82	199.95
		variance	450.37	100.65	51.21	34.52	25.42
Gamma (shape=2, scale=parameter)	(a)	mean	99.56	99.44	99.65	99.39	99.37
		variance	116.67	112.15	120.43	117.56	113.22
	(b)	mean	199.48	199.37	199.26	199.39	199.30
		variance	240.53	245.83	242.56	243.29	241.27
Lognormal (location=parameter, scale=2)	(a)	mean	89.29	98.59	98.95	98.93	99.00
		variance	5692.70	52.53	11.67	4.96	2.75
	(b)	mean	191.55	198.51	198.85	198.93	198.96
		variance	19131.99	108.18	23.68	10.47	5.78
Lognormal (scale=parameter, location=1)	(a)	mean	94.36	84.63	73.93	80.47	62.68
		variance	1608.40	70834.68	32260.12	700636.96	79507.59
	(b)	mean	194.34	181.99	176.57	167.81	168.77
		variance	3867.04	71164.76	513858.83	1669349.96	1529708.93
Weibull (shape=2, scale=parameter)	(a)	mean	100.08	100.03	100.06	100.06	100.05
		variance	3.27	3.30	3.17	3.42	3.31
	(b)	mean	200.06	200.08	200.06	200.05	200.02
		variance	6.18	6.27	6.34	6.14	6.37
Weibull (scale=1,	(a)	mean	42.30	98.70	100.07	100.18	100.17